at the Diſtance of above 120 Feet, and Silver at 50. The Theory which led me to this Diſcovery is founded upon two important Remarks, the one that the Heat is not proportional to the Quantity of Light, and the other that the Rays do not come parallel from the Sun. The firſt of thoſe, which appears to be a Paradox, is nevertheleſs a Truth, of which one may eaſily ſatisfy one's ſelf, by reflecting that Heat propagates itſelf even within Bodies; and that when one heats at the ſame time a large Superficies, the Firing is much quicker than when one only heats a ſmall Portion of the ſame.

I am, *&c.*

From the Chateau de Montbard *in* Burgundy, Sept. 18. N.S. 1748.

Buffon.

V. *An Eſſay on* Quantity; *occaſioned by reading a Treatiſe, in which* Simple *and* Compound Ratio's *are applied to* Virtue *and* Merit, *by the Rev. Mr.* Reid; *communicated in a Letter from the Rev.* Henry Miles *D.D. & F.R.S. to* Martin Folkes *Eſq; Pr.R.S.*

Sect. I.

What Quantity is.

Read Nov. 3. 1748

SINCE mathematical Demonſtration is thought to carry a peculiar Evidence along with it, which leaves no Room for further Diſpute; it may be of ſome Uſe, or Entertainment

tainment at leaſt, to inquire to what Subjects this kind of Proof may be applied.

Mathematics contain properly the Doctrine of Meaſure; and the Object of this Science is commonly ſaid to be Quantity; therefore Quantity ought to be defined, What may be meaſured. Thoſe who have defined Quantity to be whatever is capable of More or Leſs, have given too wide a Notion of it, which I apprehend has led ſome Perſons to apply mathematical Reaſoning to Subjects that do not admit of it.

Pain and Pleaſure admit of various Degrees, but who can pretend to meaſure them? Had this been poſſible, it is not to be doubted but we ſhould have had as diſtinct Names for their various Degrees, as we have for Meaſures of Length or Capacity; and a Patient ſhould have been able to deſcribe the Quantity of his Pain, as well as the Time it began, or the Part it affected. To talk intelligibly of the Quantity of Pain, we ſhould have ſome Standard to meaſure it by; ſome known Degree of it ſo well aſcertained, that all Men, when they talked of it, ſhould mean the ſame thing; we ſhould alſo be able to compare other Degrees of Pain with this, ſo as to perceive diſtinctly, not only whether they exceed or fall ſhort of it, but how far, or in what proportion; whether by an half, a fifth, or a tenth.

Whatever has Quantity, or is meaſurable, muſt be made up of Parts, which bear Proportion to one another, and to the Whole; ſo that it may be increaſed by Addition of like Parts, and diminiſhed by Subtraction, may be multiplied and divided, and in a Word, may bear any Proportion to another

Quantity of the ſame kind, that one Line or Number can bear to another. That this is eſſential to all mathematical Quantity, is evident from the firſt Elements of Algebra, which treats of Quantity in general, or of thoſe Relations and Properties which are common to all Kinds of Quantity. Every algebraical Quantity is ſuppoſed capable not only of being increaſed and diminiſhed, but of being exactly doubled, tripled, halfed, or of bearing any aſſignable Proportion to another Quantity of the ſame kind. This then is the Characteriſtic of Quantity; whatever has this Property may be adopted into Mathematics; and its Quantity and Relations may be meaſured with mathematical Accuracy and Certainty.

Sect. 2.

Of Proper and Improper Quantity.

There are ſome Quantities which may be called *Proper*, and others *Improper*. This Diſtinction is taken notice of by *Ariſtotle*; but it deſerves ſome Explication.

I call that *Proper* Quantity which is meaſured by its own Kind; or which of its own Nature is capable of being doubled or tripled, without taking in any Quantity of a different Kind as a Meaſure of it. Thus a Line is meaſured by known Lines, as Inches, Feet, or Miles; and the Length of a Foot being known, there can be no Queſtion about the Length of two Feet, or of any Part or Multiple of a Foot. And this known Length, by being multiplied or divided, is ſufficient to give us a diſtinct Idea of any Length whatſoever.

Improper Quantity is that which cannot be meaſured by its own Kind; but to which we aſſign a Meaſure by the means of ſome proper Quantity that is related to it. Thus Velocity of Motion, when we conſider it by itſelf, cannot be meaſured. We may perceive one Body to move faſter, another ſlower; but we can have no diſtinct Idea of a Proportion or *Ratio* between their Velocities, without taking in ſome Quantity of another Kind to meaſure them by. Having therefore obſerved, that by a greater Velocity a greater Space is paſſed over in the ſame time, by a leſs Velocity a leſs Space, and by an equal Velocity an equal Space; we hence learn to meaſure Velocity by the Space paſſed over in a given Time, and to reckon it to be in exact Proportion to that Space: And having once aſſigned this Meaſure to it, we can then, and not till then, conceive one Velocity to be exactly double, or half, or in any other Proportion to another; we may then introduce it into mathematical Reaſoning without Danger of Confuſion, or Error, and may alſo uſe it as a Meaſure of other Improper Quantities.

All the Kinds of Proper Quantity we know, may, I think, be reduced to theſe four, Extenſion, Duration, Number, and Proportion. Tho' Proportion be meaſurable in its own Nature, and therefore hath Proper Quantity, yet as Things cannot have Proportion which have not Quantity of ſome other Kind, it follows, that whatever has Quantity muſt have it in one or other of theſe three Kinds, Extenſion, Duration, or Number. Theſe are the Meaſures of themſelves, and of all Things elſe that are meaſurable.

Number

Number is applicable to ſome things, to which it is not commonly applied by the Vulgar. Thus, by attentive Conſideration, Lots and Chances of various Kinds appear to be made up of a determinate Number of Chances that are allowed to be equal; and by numbering theſe, the Values and Proportions of thoſe which are compounded of them may be demonſtrated.

Velocity, the Quantity of Motion, Denſity, Elaſticity, the *Vis inſita*, and *impreſſa*, the various Kinds of centripetal Forces, and different Orders of Fluxions, are all Improper Quantities; which therefore ought not to be admitted into Mathematics, without having a Meaſure of them aſſigned. The Meaſure of an improper Quantity ought always to be included in the Definition of it; for it is the giving it a Meaſure that makes it a proper Subject of mathematical Reaſoning. If all Mathematicians had conſidered, this as carefully as Sir *Iſaac Newton* appears to have done, ſome Labour had been ſaved both to themſelves and to their Readers. That Great Man, whoſe clear and comprehenſive Underſtanding appears, even in his Definitions, having frequent Occaſion to treat of ſuch improper Quantities, never fails to define them, ſo as to give a Meaſure of them, either in proper Quantities, or in ſuch as had a known Meaſure. This may be ſeen in the Definitions prefixed to his *Princip. Phil. Nat. Math.*

It is not eaſy to ſay how many Kinds of improper Quantity, may in time, be introduced into Mathematics, or to what new Subjects Meaſures may be applied: But this I think we may conclude,

that there is no Foundation in Nature for, nor can any valuable End be ſerved by applying Meaſure to any, thing but what has theſe two Properties. Firſt it muſt admit of Degrees of greater and leſs. Secondly, it muſt be aſſociated with, or related to ſomething that has proper Quantity, ſo as that when one is increaſed the other is increaſed, when one is diminiſhed, the other is diminiſhed alſo; and every Degree of the one muſt have a determinate Magnitude or Quantity of the other correſponding to it.

It ſometimes happens, that we have Occaſion to apply different Meaſures to the ſame thing. Centripetal Force, as defined by *Newton*, may be meaſured various Ways, he himſelf gives different Meaſures of it, and diſtinguiſhes them by different Names, as may be ſeen in the above-mentioned Definitions.

In reality, I conceive that the applying of Meaſures to things that properly have not Quantity, is only a Fiction or Artifice of the Mind, for enabling us to conceive more eaſily, and more diſtinctly to expreſs and demonſtrate, the Properties and Relations of thoſe things that have real Quantity. The Propoſitions contained in the two firſt Books of *Newton's Principia* might perhaps be expreſſed and demonſtrated, without thoſe various Meaſures of Motion, and of centripetal and impreſſed Forces which he uſes: But this would occaſion ſuch intricate and perplexed Circumlocutions, and ſuch a tedious Length of Demonſtrations as would fright any ſober Perſon from attempting to read them.

SECT. 3. *Coroll. firſt.*

From the Nature of Quantity we may ſee what it is that gives Mathematics ſuch Advantage over other Sciences, in Clearneſs and Certainty; namely, that Quantity admits of a much greater Variety of Relations than any other Subject of human Reaſoning; and at the ſame time every Relation or Proportion of Quantities may by the Help of Lines and Numbers be ſo diſtinctly defined, as to be eaſily diſtinguiſhed from all others, without any Danger of Miſtake. Hence it is that we are able to trace its Relations through a long Proceſs of Reaſoning, and with a Perſpicuity and Accuracy which we in vain expect in Subjects not capable of Menſuration.

Extended Quantities, ſuch as Lines, Surfaces and Solids, beſides what they have in common with all other Quantities, have this peculiar, That their Parts have a particular Place and Diſpoſition among themſelves: A Line may not only bear any aſſignable Proportion to another, in Length or Magnitude, but Lines of the ſame Length may vary in the Diſpoſition of their Parts; one may be ſtreight, another may be Part of a Curve of any Kind or Dimenſion, of which there is an endleſs Variety. The like may be ſaid of Surfaces and Solids. So that extended Quantities admit of no leſs Variety with regard to their Form than with regard to their Magnitude: And as their various Forms may be exactly defined and meaſured, no leſs than their Magnitudes, hence it is that Geometry, which treats of extended Quantity, leads us into a much greater Compaſs and Variety of Reaſoning than any other Branch of

of Mathematics. Long Deductions in Algebra for the moſt part are made, not ſo much by a Train of Reaſoning in the Mind, as by an artificial kind of Operation, which is built on a few very ſimple Principles: But in Geometry we may build one Propoſition upon another, a third upon that, and ſo on, without ever coming to a Limit which we cannot exceed. The Properties of the more ſimple Figures can hardly be exhauſted, much leſs thoſe of the more complex ones.

SECT. 4. *Coroll.* 2.

It may I think be deduced from what hath been above ſaid, That mathematical Evidence is an Evidence *ſui generis*, not competent to any Propoſition which does not expreſs a Relation of Things meaſurable by Lines or Numbers. All proper Quantity may be meaſured by theſe, and improper Quantities muſt be meaſured by thoſe that are proper.

There are many Things capable of More and Leſs, which perhaps are not capable of Menſuration. Taſtes, Smells, the Senſations of Heat and Cold, Beauty, Pleaſure, all the Affections and Appetites of the Mind, Wiſdom, Folly, and moſt Kinds of Probability, with many other Things too tedious to enumerate, admit of Degrees, but have not yet been reduced to Meaſure, nor, as I apprehend, ever can be. I ſay, moſt Kinds of Probability, becauſe one Kind of it, *viz.* the Probability of Chances is properly meaſurable by Number, as is above obſerved.

Altho' Attempts have been made to apply mathematical Reaſoning to ſome of theſe Things, and the Quantity of Virtue and Merit in Actions has been meaſured

meaſured by ſimple and compound *Ratio's*; yet I do not think that any real Knowlege has been ſtruck out this Way: It may perhaps, if diſcretely uſed, be a Help to Diſcourſe on theſe Subjects, by pleaſing the Imagination, and illuſtrating what is already known; but until our Affections and Appetites ſhall themſelves be reduced to Quantity, and exact Meaſures of their various Degrees be aſſigned, in vain ſhall we eſſay to meaſure Virtue and Merit by them. This is only to ring Changes upon Words, and to make a Shew of mathematical Reaſoning, without advancing one Step in real Knowlege.

SECT. 5. *Coroll.* 3.

I apprehend the Account that hath been given of the Nature of proper and improper Quantity may alſo throw ſome Light upon the Controverſy about the Force of moving Bodies, which long exerciſed the Pens of many Mathematicians, and for what I know is rather drop'd than ended; to the no ſmall Scandal of Mathematics, which hath always boaſted of a Degree of Evidence, inconſiſtent with Debates that can be brought to no Iſſue.

Tho' Philoſophers on both Sides agree with one another, and with the Vulgar in this, That the Force of a moving Body is the ſame, while its Velocity is the ſame, is increaſed when its Velocity is increaſed, and diminiſhed when that is diminiſhed. But this vague Notion of Force, in which both Sides agree, tho' perhaps ſufficient for common Diſcourſe, yet is not ſufficient to make it a Subject of mathematical Reaſoning: In order to that, it muſt be more accurately defined, and ſo defined as to give

us

us a Meaſure of it, that we may underſtand what is meant by a double or a triple Force. The *Ratio* of one Force to another cannot be perceived but by a Meaſure; and that Meaſure muſt be ſettled not by mathematical Reaſoning, but by a Definition. Let any one conſider Force without relation to any other Quantity, and ſee whether he can conceive one Force exactly double to another; I am ſure I cannot, nor ſhall, till I ſhall be endowed with ſome new Faculty; for I know nothing of Force but by its Effects, and therefore can meaſure it only by its Effects. Till Force then is defined, and by that Definition a Meaſure of it aſſigned, we fight in the dark about a vague Idea, which is not ſufficiently determined to be admitted into any mathematical Propoſition. And when ſuch a Definition is given, the Controverſy will preſently be ended.

Sect. 6.

Of the Newtonian Meaſure of Force.

You ſay, the Force of a Body in Motion is as its Velocity: Either you mean to lay this down as a Definition as *Newton* himſelf has done; or you mean to affirm it as a Propoſition capable of Proof. If you mean to lay it down as a Definition, it is no more than if you ſhould ſay, I call that a double Force which gives a double Velocity to the ſame Body, a triple Force which gives a triple Velocity, and ſo on in Proportion. This I intirely agree to; no mathematical Definition of Force can be given that is more clear and ſimple, none that is more agreeable to the common Uſe of the Word in Language.

For,

For ſince all Men agree, that the Force of the Body being the ſame, the Velocity muſt alſo be the ſame; the Force being increaſed or diminiſhed, the Velocity muſt be ſo alſo, what can be more natural or proper than to take the Velocity for the Meaſure of the Force?

Several other things might be advanced to ſhew that this Definition agrees beſt with the common popular Notion of the Word Force. If two Bodies meet directly with a Shock, which mutually deſtroys their Motion without producing any other ſenſible Effect, the Vulgar would pronounce, without Heſitation, that they met with equal Force; and ſo they do, according to the Meaſure of Force above laid down: For we find by Experience, that in this Caſe their Velocities are reciprocally as their Quantities of Matter. In Mechanics, where by a Machine two Powers or Weights are kept in *equilibrio*, the Vulgar would reckon that theſe Powers act with equal Force, and ſo by this Definition they do. The Power of Gravity being conſtant and uniform, any one would expect that it ſhould give equal Degrees of Force to a Body in equal Times, and ſo by this Definition it does. So that this Definition is not only clear and ſimple, but it agrees beſt with the Uſe of the Word Force in common Language, and this I think is all that can be deſired in a Definition.

But if you are not ſatisfied with laying it down as a Definition, that the Force of a Body is as its Velocity, but will needs prove it by Demonſtration or Experiment; I muſt beg of you, before you take one Step in the Proof, to let me know what you

mean by Force, and what by a double or a triple Force. This you muſt do by a Definition which contains a Meaſure of Force. Some primary Meaſure of Force muſt be taken for granted, or laid down by way of Definition; otherwiſe we can never reaſon about its Quantity. And why then may you not take the Velocity for the primary Meaſure as well as any other? You will find none that is more ſimple, more diſtinct, or more agreeable to the common Uſe of the Word Force: And he that rejects one Definition that has theſe Properties, has equal Right to reject any other. I ſay then, that it is impoſſible, by mathematical Reaſoning or Experiment, to prove that the Force of a Body is as its Velocity, without taking for granted the thing you would prove, or ſomething elſe that is no more evident than the thing to be proved.

Sect. 7.

Of the Leibnitzian *Meaſure of Force.*

Let us next hear the *Leibnitzian*, who ſays, that the Force of a Body is as the Square of its Velocity. If he lays this down as a Definition, I ſhall rather agree to it, than quarrel about Words, and for the future ſhall underſtand him, by a quadruple Force to mean that which gives a double Velocity, by 9 times the Force that which gives three times the Velocity, and ſo on in duplicate Proportion. While he keeps by his Definition, it will not neceſſarily lead him into any Error in Mathematics or Mechanics. For, however paradoxical his Concluſions may appear, however different in Words from theirs who

who meaſure Force by the ſimple *Ratio* of the Velocity; they will in their Meaning be the ſame: Juſt as he who would call a Foot twenty-four Inches, without changing other Meaſures of Length, when he ſays a Yard contains a Foot and a half, means the very ſame as you do, when you ſay a Yard contains three Feet.

But tho' I allow this Meaſure of Force to be diſtinct, and cannot charge it with Falſhood, for no Definition can be falſe, yet I ſay in the firſt place, it is leſs ſimple than the other; for why ſhould a duplicate *Ratio* be uſed where the ſimple *Ratio* will do as well? In the next place, this Meaſure of Force is leſs agreeable to the common Uſe of the Word Force, as hath been ſhewn above; and this indeed is all that the many laboured Arguments and Experiments, brought to overturn it, do prove. This alſo is evident, from the Paradoxes into which it has led its Defenders.

We are next to conſider the Pretences of the *Leibnitzian*, who will undertake to prove by Demonſtration, or Experiment, that Force is as the Square of the Velocity. I ask him firſt, what he lays down for the firſt Meaſure of Force? The only Meaſure I remember to have been given by the Philoſophers of that Side, and which ſeems firſt of all to have led *Leibnitz* into his Notion of Force, is this: The Height to which a Body is impell'd by any impreſſed Force, is, ſays he, the whole Effect of that Force, and therefore muſt be proportional to the Cauſe: But this Height is found to be as the Square of the Velocity which the Body had at the Beginning of its Motion.

In this Argument I apprehend that great Man has been extremely unfortunate. For, *1ſt*, Whereas all Proof ſhould be taken from Principles that are common to both Sides, in order to prove a thing we deny, he aſſumes a Principle which we think farther from the Truth; namely, that the Height to which the Body riſes is the whole Effect of the Impulſe, and ought to be the whole Meaſure of it. *2dly*, His Reaſoning ſerves as well againſt him as for him: For may I not plead with as good Reaſon at leaſt thus? The Velocity given by an impreſſed Force is the whole Effect of that impreſſed Force; and therefore the Force muſt be as the Velocity. *3dly*, Suppoſing the Height to which the Body is raiſed to be the Meaſure of the Force, this Principle overturns the Concluſion he would eſtabliſh by it, as well as that which he oppoſes. For, ſuppoſing the firſt Velocity of the Body to be ſtill the ſame; the Height to which it riſes will be increaſed, if the Power of Gravity is diminiſhed; and diminiſhed, if the Power of Gravity is increaſed. Bodies deſcend ſlower at the Equator, and faſter towards the Poles, as is found by Experiments made on Pendulums. If then a Body is driven upwards at the Equator with a given Velocity, and the ſame Body is afterwards driven upwards at *Leipſick* with the ſame Velocity, the Height to which it riſes in the former Caſe will be greater than in the latter; and therefore, according to his Reaſoning, its Force was greater in the former Caſe; but the Velocity in both was the ſame; conſequently the Force is not as the Square of the Velocity any more than as the Velocity.

SECT.

Sect. 8.

Reflections on this Controverſy.

Upon the whole, I cannot but think the Controvertiſts on both Sides have had a very hard Task; the one to prove, by mathematical Reaſoning and Experiment, what ought to be taken for granted, the other by the ſame means to prove what might be granted, making ſome Allowance for Impropriety of Expreſſion, but can never be proved.

If ſome Mathematician ſhould take it in his Head to affirm, that the Velocity of a Body is not as the Space it paſſes over in a given Time, but as the Square of that Space; you might bring mathematical Arguments and Experiments to confute him; but you would never by theſe force him to yield, if he was ingenuous in his Way; becauſe you have no common Principles left you to argue from, and you differ from one another, not in a mathematical Propoſition, but in a mathematical Definition.

Suppoſe a Philoſopher has conſider'd only that Meaſure of centripetal Force which is proportional to the Velocity generated by it in a given Time, and from this Meaſure deduces ſeveral Propoſitions. Another Philoſopher in a diſtant Country, who has the ſame general Notion of centripetal Force, takes the Velocity generated by it, and the Quantity of Matter together, as the Meaſure of it. From this he deduces ſeveral Concluſions, that ſeem directly contrary to thoſe of the other. Thereupon a ſerious Controverſy is begun, whether centripetal Force be as the Velocity, or as the Velocity and Quantity of Matter taken together. Much mathematical and

experimental Duſt is raiſed; and yet neither Party can ever be brought to yield; for they are both in the right, only they have been unlucky in giving the ſame Name to different mathematical Conceptions. Had they diſtinguiſhed theſe Meaſures of centripetal Force as *Newton* has done, calling the one *Vis centripeta Quantitatis acceleratrix*, the other *Quantitas motrix*; all Appearance of Contradiction had ceaſed, and their Propoſitions, which ſeem ſo contrary, had exactly tallied.

VI. *A Letter from* Rich. Haſſel *Eſq*; *F.R.S. to* Peter Daval *Eſq*; *Secr. R. S. concerning a large Piece of a* Lath *being thruſt into a Man's* Eye, *who recover'd of it.*

SIR,

Read Nov. 10. 1748. I THOUGHT the following Caſe ſo extraordinary as to be worth the Notice of the *Royal Society*. If you think ſo too, I beg you to communicate it.

On *Sunday* the 17th of *Jan.* 1747. *Henry Halſey*, of *South Mims*, Labourer, thruſt a long Lath with great Violence into the great *Canthus* of the left Eye of *Edward Roberts* of the ſame Place Labourer, which broke off quite ſhort; ſo that a Piece two Inches and near a half long, half an Inch wide, and above a quarter of an Inch thick, (*ſee* TAB. I.) remained in his Head, and was ſo deeply buried there, that it could ſcarce be